Cambridge Assessment International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics 33 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1	$\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(\cos\alpha + \cos\theta)$	**M1**	Energy equation, 4 terms, dimensionally correct, mass must be present, allow sign errors, allow sin in both terms on RHS
	$mg\cos\theta = \frac{mv^2}{a}$	**B1**	N2L, may include tension initially but not awarded until tension = 0 used
	$\frac{3}{2}mag - \frac{1}{2}m.ag\cos\theta = mga\left(\frac{4}{5} + \cos\theta\right)$ $\frac{3}{2}\cos\theta = \frac{7}{10}$	**M1**	Dependent on tension = 0 and on an energy equation, eliminate v^2 .
	$\cos\theta = \frac{7}{15}$	**A1**	If no *m* in energy equation and no further errors, award SCB2 for correct final answer
		4	

Question	Answer	Marks	Guidance
2	$\frac{1}{2}m\left(v^2-\frac{v^2}{4}\right)=\frac{\frac{1}{2}\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2-\left(\frac{1}{3}a\right)^2\right)$	M1	Kinetic energy = elastic potential energy, 4 terms, dimensionally correct, allow sign errors.
	$\frac{1}{2}m\left(2ag-\frac{1}{2}ag\right)=\frac{\frac{1}{2}\lambda mg}{a}\left(\left(\frac{2}{3}a\right)^2-\left(\frac{1}{3}a\right)^2\right)$	A1	With v substituted.
	Solve [$\frac{3}{4}v^2=\lambda g\left(\frac{3}{9}a\right)$]	M1	Solve to find value for λ dependent on energy equation with 3 or 4 terms
	$\lambda=\frac{9}{2}$	A1	SCB2 for $\lambda=\frac{9}{2}mg$ if given λ not used
		4	

Question	Answer	Marks	Guidance
3(a)	[Mass is proportional to area] <table><tr><td></td><td>Area</td><td>Centre of mass from AC</td></tr><tr><td>ABC</td><td>$\frac{1}{2}.6a.8a \ (=24a^2)$</td><td>$2a$</td></tr><tr><td>$DEC$</td><td>$\frac{1}{2}x.5a$</td><td>$\frac{1}{3}x$</td></tr><tr><td>$ADEB$</td><td>$24a^2 - \frac{5}{2}xa$</td><td>$\overline{x}$</td></tr></table>	**B1**	All correct for ABC and DEC.
	Moments [about AC] $\overline{x}\left(24a^2 - \frac{5}{2}xa\right) = 24a^2 \times 2a - \frac{1}{3}x \times \frac{5}{2}ax$	**M1**	All moment terms present, dimensionally correct, allow sign error.
		A1	All correct moments about AC.
	$\overline{x} = \frac{288a^2 - 5x^2}{3(48a - 5x)}$	**A1**	AEF
		4	

Question	Answer	Marks	Guidance
3(b)	On the point of toppling about *E*: $\bar{x} = x$, $\dfrac{288a^2 - 5x^2}{3(48a - 5x)} = x$	**B1 FT**	FT *their* expression for $\bar{x}$ from part **(a)**.
	Rearrange to 3-term quadratic: $10x^2 - 144ax + 288a^2 = 0$	**M1**	Allow 3-term inequality.
	$2(5x - 12a)(x - 12a) = 0$, $x = \dfrac{12}{5}a$	**A1**	Single correct answer, no inequality, CWO.
		3	

Question	Answer	Marks	Guidance
4(a)	Let speeds of A and B along line of centres after collision be V_A and V_B $V_A + V_B = u\cos 30°$ (1)	**M1**	Allow sign errors, allow missing m.
	$-V_A + V_B = eu\cos 30°$ (2)	**M1**	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$ Moving in same direction, so $\dfrac{V_A}{u\sin 30°} = \dfrac{V_B}{2u}$ (3)	**B1**	SOI $V_B = 4V_A$
	Use $V_B = 4V_A$ in (1): $5V_A = u\cos 30°$ From (2): $3V_A = eu\cos 30°$ then Combine to find equation in e only.	**M1**	A complete method to find equation in e only
	$e = \frac{3}{5}$	**A1**	

Question	Answer	Marks	Guidance
4(a)	**Alternative method for question 4(a)**		
	Let speeds of A and B along line of centres after collision be V_A and V_B $V_A + V_B = u\cos 30°$ (1)	M1	Allow sign errors, allow missing m.
	$-V_A + V_B = eu\cos 30°$ (2)	M1	Signs on LHS must be consistent with (1).
	Speeds perpendicular to line of centres after collision are $u\sin 30°$ and $2u$ Moving in same direction, so $\frac{V_A}{u\sin 30°} = \frac{V_B}{2u}$ (3)	B1	SOI $V_B = 4V_A$
	Solve (1) and (2): $V_A = \frac{1}{2}u(1-e)\cos 30°$, $V_B = \frac{1}{2}u(1+e)\cos 30°$ Substitute in (3) to find equation in e only .	M1	Note: $V_A = \frac{u}{10}\sqrt{3}$, $V_B = \frac{4u}{10}\sqrt{3}$
	$e = \frac{3}{5}$	A1	
		5	
4(b)	KE after = $\frac{1}{2}m\left(V_A^2 + \left(\frac{u}{2}\right)^2\right) + \frac{1}{2}m\ ((2u)^2 + V_B^2\ )$	B1	Correct expression for KE for one of the spheres, after collision, with both components.
	KE for A after = $\frac{7}{50}mu^2$ or KE for B after = $\frac{56}{25}mu^2$ or KE loss for A = $\frac{9}{25}mu^2$ or KE **gain** for B = $\frac{6}{25}mu^2$	B1	Implied by total KE after = $\frac{119}{50}mu^2$.
	Total loss in KE = $\frac{3}{25}mu^2$	B1	Term $\frac{1}{2}m(2u)^2$ may be omitted from KE of B before and after.
		3	

Question	Answer	Marks	Guidance
5(a)	$\uparrow T\cos\theta = mg$	B1	
	$\rightarrow T\sin\theta = \left[\dfrac{mv^2}{r} =\right] m \times \dfrac{27ag}{4r}$	B1	
	$r = 12a\tan\theta$ used	M1	
	Divide: $\tan\theta = \dfrac{27}{4\times 12\tan\theta}$, so $\tan\theta = \dfrac{3}{4}$	M1	Finds value for $\tan\theta$ OE. Reduces to equation in θ or x, no k.
	$r = 9a$, extension of string $= 3a$	A1	
	Alternative method for question 5(a)		
	Let L be stretched length of string. $\uparrow T\cos\theta = mg$	B1	Or $T \times \dfrac{12a}{L} = mg$
	$\rightarrow T\sin\theta = \left[\dfrac{mv^2}{r} =\right] m \times \dfrac{27ag}{4r}$	B1	
	$r = L\sin\theta$ used	M1	
	Use $\cos\theta = \dfrac{12a}{L}$ and $\sin\theta = \dfrac{\left(L^2 - 144a^2\right)^{0.5}}{L}$ and eliminate T.	M1	
	$[L^2 - 144a^2 = 81a^2]$ $L = 15a$, extension of string $= 3a$	A1	
		5	

Question	Answer	Marks	Guidance
5(b)	Hooke's law: $T = \frac{kmg(L-12a)}{12a}$	**B1**	
	Eliminate T: $\frac{kmg(L-12a)}{12a} = \frac{mgL}{12a}$	**M1**	
	$k = \frac{L}{L-12a} = 5$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	$m\frac{dv}{dt} = mg(1-kv)$	**B1**	Mass must be seen at this point or earlier. [SUVAT does **not** apply.]
	$-\frac{1}{k}\ln(1-kv) = gt + A$	**M1**	Separate variables and integrate to logarithm.
		A1	Correct, with constant of integration.
	$t = 0, v = 0$ $[A = 0]$	**M1**	Use initial condition to evaluate their constant.
	$v = \frac{1}{k}\left(1-e^{-kgt}\right)$	**A1**	Any correct form with v as subject. Final A0 if numerical value of g present.
		5	

Question	Answer	Marks	Guidance
6(b)	$k = 0.05$ and so $\frac{dx}{dt} = 20\left(1 - e^{-0.5t}\right)$	*M1	Attempt to integrate if expression contains a term of the form be^{ct} .
	Integrate: $x = 20\left(t + 2e^{-0.5t}\right) + B$	A1	$x = \frac{1}{k}\left(t + \frac{1}{gk}e^{-kgt}\right) + B$
	$t = 0,\ x = 0\ \left[B = -40\right]$	DM1	Use initial condition to evaluate their constant.
	When $v = 12$, from part **(a)**, $e^{-0.5t} = 1 - 0.05 \times 12 = 0.4$, $t = -2\ln 0.4$	M1	1.83…
	$x = -40\ln 0.4 + 40 \times 0.4 - 40 = 12.7$	A1	$40\ln\frac{5}{2} - 24$
	Alternative method for question 6(b)		
	$v\frac{dv}{dx} = g(1 - kv)$ leading to $\left(1 - \frac{1}{1 - kv}\right)dv = -kgdx$	*M1	Separate variables and write in integrable form
	$v + \frac{1}{k}\ln(1 - kv) = -kgx + B$	DM1 A1	Dependent on previous M1. Attempt to integrate.
	$v = 0,\ x = 0\ \left[B = 0\right]$ and $k = 0.05,\ v = 12\ \ 12 + 20\ln 0.4 = -0.5x$	M1	Dependent on both previous M1s. Use initial condition to evaluate their constant and use $v = 12$
	$x = 12.7$	A1	$40\ln\frac{5}{2} - 24$
		5	

Question	Answer	Marks	Guidance
7(a)	For aircraft, $d = 5T$	**B1**	
	For ball, $\uparrow 4 = u\sin\theta\, T - \frac{1}{2}\times 10\times T^2$	**B1**	To point of collision.
	For ball, $\rightarrow u\cos\theta\, T = d + 8 = 5T + 8$	**B1**	
	Eliminate u: $4 = \frac{u4}{5}T - \frac{1}{2}\times 10\times T^2,\quad u = \frac{5\left(4+5T^2\right)}{4T}$ and $u = \frac{5(5T+8)}{3T}$ $3\left(4+5T^2\right) = 4(5T+8)$	***M1**	Dependent on LHS of second B1 being 4, expression involving only T
	$3T^2 - 4T - 4 = 0$	**DM1**	Dependent on previous M1. Obtain and solve 3-term quadratic.
	$T = 2$	**A1**	Single correct answer.
		6	Note $d - 8$ used leads to $T = \frac{2}{3}$ B1B1B0M1M1A0

Question	Answer	Marks	Guidance
7(b)	$\tan^{-1}\dfrac{(u\sin\theta - 10T)}{u\cos\theta}$	**M1**	OE Accept 'tan = ……'
	$\tan^{-1}\dfrac{8}{9}$	**A1**	OE
	Direction is 41.6° below the horizontal	**A1**	CAO Note: $d-8$ used leads to $20.9°$ above the horizontal.
		3	